

Non-maximality and vagueness: Revisiting the plural Sorites paradox*

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Abstract This paper is an attempt at a synthesis of two superficially conflicting approaches to non-maximality: the issue-based approach (Malamud 2012; Križ 2015; Križ & Spector 2021 a.o.), which generates clear-cut truth conditions once the issue parameter has been fixed, and the strict/tolerant approach (Burnett 2017 a.o.), on which non-maximal construals involve vagueness. I argue that there are two classes of contexts that license non-maximality. One of them gives rise to the Sorites paradox once the non-embeddability of non-maximality is controlled for. The other class does not license vagueness at all. To model this distinction, I introduce a formal framework that combines the issue-based approach with the notion of strict and tolerant truth conditions (Cobrerros, Egré, Ripley & van Rooij 2012a), which are defined via super-/subvaluation over different issues. This system provides two sources of non-maximality, only one of which involves vagueness.

Keywords: plurals, non-maximality, vagueness, strict/tolerant truth, Sorites paradox, QUD

1 Introduction

Definite plurals are traditionally taken to pick out the maximal plural individual in the domain provided by the NP (Sharvy 1980; Link 2002 [1983] a.o.). On this approach, sentences like (1a) receive the same truth conditions as universally quantified sentences like (1b).¹ But in many contexts this semantics, exemplified

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¹ Let me briefly recapitulate some basic notions of plural semantics (see e.g. Link 2002 [1983]; Schwarzschild 1996 a.o.) We take the individual domain D_e to be closed under a **sum operation** \oplus that maps any subset of D_e to its sum. We further assume that there is a set $A \subset D_e$ of **atomic individuals** such that the elements in D_e stand in a one-to-one correspondence to nonempty subsets of A , i.e. the structures (D_e, \oplus) and $(\mathcal{P}(A) \setminus \{\emptyset\}, \cup)$ are isomorphic. In addition, I use the following notational conventions: (i) $x \oplus y := \oplus(\{x, y\})$; (ii) $x \leq y := x \oplus y = y$; (iii) $x \leq_a y := x \leq y \wedge x \in A$.

in (1c), is too strong to capture the conditions under which we actually accept definite plural sentences (henceforth referred to as **pragmatic truth conditions**). For instance, in scenario (2), (1a) seems acceptable, while (1b) seems false.

- (1) a. *The windows are open.* ✓ in (2)
 b. *All the windows are open.* × in (2)
 c. $\lambda w. *open_w(\oplus(\{x \mid \mathbf{windows}_w(x)\}))$
 (where $*(P)(x)$ iff $\exists A. \oplus A = x \wedge \forall y \in A. P(y)$; see Link 2002 [1983] a.o.)
- (2) DRYING WALLS: Ann and Sue have been renovating the walls of a lecture hall with 50 windows. For the material to dry quickly enough, the room has to be well ventilated. Sue asks how good the ventilation in the room is. Ann can see that 45 of the windows are open, while the remaining 5 are closed.

This phenomenon is known as **non-maximality**², and I will refer to contexts in which a definite plural sentence is acceptable while the corresponding *all*-sentence is unacceptable as **non-maximal contexts**. The context-dependency of non-maximal construals is at the core of a recent group of analyses (Malamud 2012; Križ 2015, 2016; Križ & Spector 2021; Bar-Lev 2021; Feinmann 2020; see already Krifka 1996 for the intuition) that relate their availability to a contextually provided **issue**, i.e. a partition of the logical space (Groenendijk & Stokhof 1984).

Such approaches, which I will refer to as **issue-based theories**, are motivated by the observation that non-maximal contexts involve a particular kind of Question Under Discussion (QUD). For instance, the DRYING WALLS scenario arguably evokes a QUD for which it is irrelevant whether all 50 windows or just 45 of them are open, so that these two types of worlds are in the same partition class. Issue-based theories take definite plural sentences to be context-dependent, but given a fixed context, their pragmatic truth conditions are predicted to be clear-cut. This seems to conflict with another recent approach to non-maximality, spelled out most explicitly in Burnett 2017, on which non-maximality inherently involves vagueness.³ This paper argues that plural sentences exhibit vagueness in some, but not all contexts. Hence, the two approaches are in fact compatible and account for distinct subsets of the data. Further, the distinction between vague and non-vague non-maximal contexts can be modelled in terms of the issue parameter and its relation to the overt QUD. I therefore propose a synthesis of the two approaches that combines Križ &

² See Fodor 1970; Schwarzschild 1994; Krifka 1996; Yoon 1996; Landman 1996; Brisson 1998; Lasersohn 1999; Malamud 2012; Križ 2015, 2016; Burnett 2017; Križ & Spector 2021; Feinmann 2020; Bar-Lev 2021 a.o. for relevant work.

³ Some recent work on non-maximality does not fit either of these categories, e.g. the probabilistic approach recently developed by Feinmann (2020). For reasons of space, a discussion of the differences between his proposal and my non-probabilistic approach to vagueness is deferred to future work.

Spector’s (2021) version of the issue-based theory with the **strict/tolerant** approach to vagueness (Cobrerros, Egré, Ripley & van Rooij 2012b; Cobrerros et al. 2012a).

I will start by introducing Križ & Spector’s (2021) issue-based theory in detail (Section 2) and spelling out Burnett’s (2017) argument for vagueness in non-maximal predication (Section 3). In Section 4, I argue that vagueness arises only in contexts with a special property: There must be several potential values for the issue parameter, which stand in a non-transitive similarity relation. In the subsequent sections, I formalize this idea within a variant of the strict/tolerant approach to vagueness that involves super- and subvaluation (Cobrerros et al. 2012a). Section 5 introduces this framework using the example of *tall*, Section 6 extends it to non-maximality and Section 7 concludes the paper. While the present proposal is obviously indebted to Burnett’s (2017) previous analysis of plurals in the strict/tolerant framework, a detailed comparison of the two analyses is left to future work for reasons of space.

2 Issue-based theories of non-maximality

This section lays out some of the basic predictions of issue-based theories, based on a simplified version of Križ & Spector’s (2021) framework. I will assume that a context C provides a **valuation function** v_C that maps contextual parameters to their values, and write $v_C(I)$ for the value of the issue parameter I in C . Further, I write $\llbracket \phi \rrbracket^{v,w}$ for the semantic value of ϕ relative to a valuation function v and world w , and $\llbracket \phi \rrbracket_p^{v,w}$ for its pragmatic truth value relative to v and w .

For sentences without plurals, $\llbracket \phi \rrbracket_p^{v,w} = \llbracket \phi \rrbracket^{v,w}$. But plural sentences are semantically underspecified: If ϕ contains a definite plural, $\llbracket \phi \rrbracket^{v,w}$ is not a truth value, but a set of propositions corresponding to the possible maximal and non-maximal construals, i.e. $\llbracket \phi \rrbracket^{v,w}$ is of type $\langle \langle s, t \rangle, t \rangle$ rather than t . In the case of (1a), each proposition in $\llbracket \phi \rrbracket^{v,w}$ quantifies existentially over a set of subpluralities of the windows. The full set $\llbracket \phi \rrbracket^{v,w}$ can then be characterized as in (3), where each proposition involves a different function S that maps any individual x to an upward-closed set of parts of x .⁴ The different choices of S correspond to different ‘degrees’ of non-maximality: If S returns the singleton set $\{\oplus(\{x \mid \mathbf{windows}_w(x)\})\}$, the result is a maximal interpretation corresponding to the *all*-sentence in (1b); if S returns smaller subpluralities of the windows as well, we get a construal weaker than (1b). However, many of these construals will still be stronger than an indefinite plural sentence with *some of the windows* since S might not return all parts of the window plurality.

$$(3) \quad \llbracket \textit{The windows are open} \rrbracket^{v,w} = \{ \lambda w'. \exists y \in S(\oplus(\{x \mid \mathbf{windows}_w(x)\})) . * \mathbf{open}_{w'}(y) \mid S: D_e \rightarrow \mathcal{P}(D_e) \wedge \forall x \in D_e [S(x) \text{ is an upward-closed set of parts of } x] \}$$

⁴ A set A is an **upward-closed set of parts** of an individual x iff (i) all elements of A are parts of x , (ii) $x \in A$ and (iii) for any individual $y \leq x$ such that $y \in A$, any z such that $y \leq z \leq x$ is also in A .

The pragmatic truth conditions of a plural sentence ϕ in a context C depend on a subset of $\llbracket \phi \rrbracket^{v_C, w}$ determined by the issue $v_C(I)$. The scenarios typically used to exemplify this involve clear-cut binary decisions. Consider the BANK ROBBERY scenario in (4a) (adapted from Krifka 1996). Here the pragmatic truth conditions of (4b) seem to be that a subset of the doors that provides a path to the safe was open. This proposition, given in (5), is an element of $\llbracket \textit{The doors are open} \rrbracket^{v, w}$, obtained by choosing S such that it returns only those subpluralities that form a path to the safe.

- (4) a. BANK ROBBERY: A bank vault is accessible via a corridor with several doors as pictured in Figure 1. Ann and Sue wanted to steal the safe. They bribed someone to give Ann access to the vault, but in the end their plan was unsuccessful. Sue wants to know how it went.
 b. Ann: *The doors were open (but I was stopped by the security guard).*
 \checkmark in situation (A) in Figure 1, \times in situation (B) in Figure 1
- (5) $\lambda w. \exists y \in S(\bigoplus(\{x \mid \mathbf{doors}_w(x)\})). * \mathbf{open}_{w'}(y)$
 where $S(d_1 \oplus d_2 \oplus d_3 \oplus d_4) = \{d_1 \oplus d_2 \oplus d_3, d_1 \oplus d_2 \oplus d_4, d_1 \oplus d_2 \oplus d_3 \oplus d_4\}$

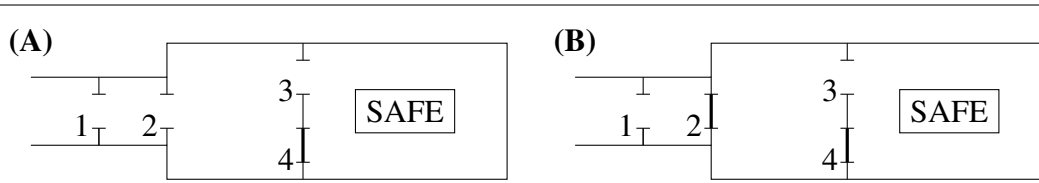


Figure 1 BANK ROBBERY scenario (see (4a))

Implementations of the issue-based approach differ in the way the underspecified meanings of plural sentences are mapped to pragmatic truth conditions. Abstracting away from the technicalities, the idea in Kríž & Spector 2021 is that the pragmatic truth value of a plural sentence ϕ under a valuation function v , $\llbracket \phi \rrbracket_{\mathbf{p}}^{v, w}$, is determined only by those propositions from $\llbracket \phi \rrbracket^{v, w}$ that divide the logical space along the lines specified by the issue $v(I)$. Specifically, $\llbracket \phi \rrbracket_{\mathbf{p}}^{v, w}$ is derived via universal quantification over those propositions from $\llbracket \phi \rrbracket^{v, w}$ that are **strongly relevant** to $v(I)$:

- (6) A proposition p is **strongly relevant** to an issue Q iff it is the disjunction of a non-empty proper subset of partition cells of Q .
- (7) Given a valuation function v :
- $\llbracket \phi \rrbracket_{\mathbf{p}}^{v, w} = 1$ iff $\forall p [p \in \llbracket \phi \rrbracket^{v, w} \wedge p$ strongly relevant to $v(I) \rightarrow p(w) = 1]$
 - $\llbracket \phi \rrbracket_{\mathbf{p}}^{v, w} = 0$ iff $\forall p [p \in \llbracket \phi \rrbracket^{v, w} \wedge p$ strongly relevant to $v(I) \rightarrow p(w) = 0]$

Two aspects of this definition are noteworthy. First, a proposition may fail to be strongly relevant if it is overinformative. In scenario (4a), the salient ‘issue’ is the yes/no question illustrated in (8): ‘Was there a subset of open doors that formed a path to the safe?’ Given this choice of $v_C(I)$, (5) is the only strongly relevant proposition in $\llbracket \phi \rrbracket^{vc,w}$; note that while the maximal construal (‘ d_1, d_2, d_3 and d_4 were all open’) is relevant in the standard Gricean sense, it is a proper subset of a partition cell in (8) and therefore not strongly relevant. So the pragmatic truth and falsity conditions of (4b) in this scenario are fully determined by the proposition (5).

- (8) Issue in the BANK ROBBERY scenario (where w_S is a world in which exactly the doors in S are open)

w_\emptyset	$w_{\{d_1\}}$	$w_{\{d_2\}}$	$w_{\{d_1,d_2\}}$	$w_{\{d_1,d_2,d_4\}}$
$w_{\{d_1,d_3\}}$	$w_{\{d_2,d_3\}}$	$w_{\{d_3\}}$	$w_{\{d_1,d_4\}}$	$w_{\{d_2,d_4\}}$
$w_{\{d_4\}}$	$w_{\{d_3,d_4\}}$	$w_{\{d_1,d_3,d_4\}}$	$w_{\{d_2,d_3,d_4\}}$	$w_{\{d_1,d_2,d_3,d_4\}}$

Second, whenever multiple propositions in $\llbracket \phi \rrbracket^{vc,w}$ are strongly relevant, a gap emerges between the pragmatic truth and falsity conditions of ϕ . Consider the question ‘Which, if any, of the doors were open?’, which puts any two worlds in (8) into distinct partition cells. Then all the propositions in (3) are strongly relevant, so that for ϕ to be pragmatically true, they must all be true—including the universal construal in (9a)—and for ϕ to be pragmatically false, they must all be false—including the existential construal in (9b). So if some, but not all of the doors are open, ϕ is pragmatically neither true nor false. Such **homogeneity** effects are well-studied for plurals in the absence of explicit QUDs (see Fodor 1970; Löbner 2000; Schwarzschild 1994; Gajewski 2005; Križ 2015; Križ & Chemla 2015; Bar-Lev 2021 a.o.). The question arises how these gaps relate to borderline cases of vague predicates—e.g. our reluctance to accept or reject (10) if Peter’s height is average.

- (9) a. $\lambda w'. * \text{open}_{w'}(\bigoplus(\{x \mid \text{doors}_w(x)\}))$ (S returns $\{\bigoplus(\{x \mid \text{doors}_w(x)\})\}$)
 b. $\lambda w'. \exists y. \text{doors}_w(y) \wedge * \text{open}_{w'}(y)$ (S returns set of all NP subpluralities)
- (10) *Peter is tall (for a German man).*

Most of the issue-based literature takes homogeneity and vagueness to be unrelated, based on arguments that homogeneity gaps do not reduce to borderline cases (see Križ 2015; Feinmann 2022). But even if we accept these arguments, non-maximal construals could still involve vagueness—an issue to which we now turn.

3 What exactly is vague about plural predication?

If plurals induce truth-value gaps unrelated to vagueness, the crucial diagnostic for vague plural predication should be not the presence of gaps, but the Sorites paradox.

Thus, (10) is vague because it gives rise to the paradoxical reasoning in (11): Premise (11a) seems clearly true, and each instance of the schema in (11b) seems acceptable too. But then how do we avoid the conclusion in (11c)? Note that the challenge is not just to block this inference, but to simultaneously account for the plausibility of the premises (see Raffman 1996; Graff 2000; Cobreros et al. 2012b a.o.).

- (11) a. *If Peter's height is 185cm, Peter is tall (for a German man).*
 b. For $175 < n \leq 185$, the following inference holds:
If Peter's height is n cm, Peter is tall (for a German man). \models *If Peter's height is $n - 1$ cm, Peter is tall (for a German man).*
 c. \therefore *If Peter's height is 175cm, Peter is tall (for a German man).*

In what sense does the paradox extend to non-maximality? Burnett (2017: 152) illustrates the phenomenon using scenarios like (12a). As she notes, (12b) seems clearly false if less than 50% of the townspeople are asleep, but what about, say, 75%? One cannot pinpoint the maximum number of 'exceptions' that we would be willing to tolerate, and "subtracting a single townspeople" cannot take us from a scenario in which (12b) is fully acceptable to one where it is clearly unacceptable.

- (12) a. HOMETOWN: The speaker is describing what her hometown is like at night. It is a quiet town with no nightlife. Typically, a few people are taking walks or quietly watching TV; everyone else is asleep.
 b. \checkmark *The townspeople are asleep.*

How can we turn this into a Sorites argument? The most natural version involves metalinguistic reasoning about the acceptability of (12b), as in (13).⁵ Again, the puzzle is how to reconcile the plausibility of the premises in (13b) with the invalidity of an inference from (13a) and (13b) to (13c). Note that it is not easy to formulate the paradox in the object language, since conditionals like (14) are somewhat odd.

- (13) a. *If all the townspeople are asleep, **The townspeople are asleep** is acceptable.*

⁵ I used the metalinguistic predicate *acceptable* rather than *true* here to sidestep the judgment, widely shared in the literature, that plural sentences are never really true in a non-maximal scenario (see e.g. Lasersohn 1999). In fact, Feinmann (2020: 54) claims on the basis of an example analogous to (13), but with the predicate *true*, that there is no Sorites paradox in plural predication. But the metalinguistic predicate that exhibits vagueness is not a context-independent notion of truth, but the property I called 'pragmatically true' in the main text (called 'true enough' in Križ 2015). In this context, it is interesting that Križ & Chemla (2015) report that to elicit clear-cut judgments targeting the maximal construal of definite plural sentences in a trivalent task, they had to name the three options *completely true*, *completely false* and *neither* rather than *true*, *false* and *neither*. This suggests that for linguistically/philosophically naive speakers, the predicate *true* can target 'pragmatic truth'. My claims in this paper then predict that for such speakers, *true* is a vague predicate.

- b. For any $n > 2$, the following inference holds:
*If n of the townspeople are asleep, **The townspeople are asleep** is acceptable. \models If $n - 1$ of the townspeople are asleep, **The townspeople are asleep** is acceptable.*
- c. \therefore *If 2 of the townspeople are asleep, **The townspeople are asleep** is acceptable.*

(14) *?If n of the townspeople are asleep, the townspeople are asleep.*

The reason for the low acceptability of (14) appears to be that the mapping from the semantic value $\llbracket \phi \rrbracket^{v,w}$ to the pragmatic truth value $\llbracket \phi \rrbracket_{\mathbf{p}}^{v,w}$ (see definition (7)) is not part of the semantic composition and therefore cannot be embedded under a conditional. This non-embeddability is part of a broader divide between vagueness proper and imprecision phenomena such as non-maximality. The prevailing view is that these two classes of phenomena should receive independent accounts (see e.g. Kennedy 2007; Lasersohn 1999).⁶ But this view fails to explain why they are both susceptible to the Sorites paradox once constraints on embedding are controlled for. I therefore follow Burnett (2017) in taking the Sorites paradox, rather than a particular embedding pattern, to be indicative of vagueness and pursuing a uniform approach to Sorites arguments involving *tall* and those involving definite plurals. On this view, the reason why the plural version of the paradox requires a metalinguistic predicate like *acceptable* is that such predicates have a semantics similar to the pragmatic truth definition in (7), but are embeddable. Since an adequate analysis of such predicates is beyond the scope of this paper, I will assume the schema in (15):

$$(15) \quad \llbracket \phi \text{ is acceptable} \rrbracket^{v,w} = 1 \text{ iff } \llbracket \phi \rrbracket_{\mathbf{p}}^{v,w} = 1 \\ = 1 \text{ iff } \forall p [p \in \llbracket \phi \rrbracket^{v,w} \wedge p \text{ strongly relevant to } v(I) \rightarrow p(w) = 1]$$

4 Distinguishing vague from non-vague non-maximal contexts

We have seen that definite plurals can give rise to the Sorites paradox. Under which conditions does this happen? As the DRYING WALLS scenario in (16a) shows, vagueness can arise even in the presence of an explicit QUD: One cannot pinpoint the minimum number of open windows that would still make (16b) acceptable. So the DRYING WALLS scenario is susceptible to the Sorites argument in (17) while the BANK ROBBERY scenario is not, even though both involve explicit QUDs.

- (16) a. DRYING WALLS: Ann and Sue have been renovating the walls of a lecture hall with 50 windows. For the material to dry quickly enough, the room has to be well ventilated. Sue asks how good the ventilation in the room is.

⁶ Thanks to Clemens Steiner-Mayr and Diego Feinmann for bringing up this issue.

- b. Ann: *Well, the windows are open.* ✓ if 48/50 windows are open
 × if 2/50 windows are open
- (17) a. *If all the windows are open, The windows are open is acceptable.*
- b. For any $n > 2$, the following inference holds:
 *If n of the windows are open, The windows are open is acceptable. ⊨ If
 $n - 1$ of the windows are open, The windows are open is acceptable.*
- c. ∴ *If 2 of the windows are open, The windows are open is acceptable.*

A common reaction to scenarios like DRYING WALLS is that they involve inherently ‘vague’ QUDs: It is simply not clear whether 40 open windows out of 50 would count as ‘good ventilation’ (see Graff 2000 for similar intuitions). This is a plausible idea, but not directly expressible if issues are modeled as partitions of the logical space. Consider a sequence of worlds w_n , $0 \leq n \leq 50$, such that n of the windows are open in w_n . Then in order to be able to model the QUD in (16) as a partition, one has to decide whether the room counts as well ventilated in, say, w_{45} . If so, and if the partition $\nu_C(I)$ separates w_{45} from w_{44} , the Križ & Spector 2021 framework predicts the sentence to be pragmatically true in w_{45} , but clearly not pragmatically true in w_{44} . This prediction of a clear-cut boundary seems correct for the BANK ROBBERY scenario, where the sentence is clearly acceptable given situation (A) in Figure 1 and unacceptable given situation (B). But in the DRYING WALLS scenario, it is never the case that the sentence is clearly acceptable in w_{i+1} and clearly unacceptable in w_i .

I submit that the difference between vague non-maximal contexts like DRYING WALLS or HOMETOWN and non-vague ones like BANK ROBBERY is better described in terms of the relation between the issue parameter and the intuitively salient QUD. On most versions of the issue-based approach to non-maximality, these are distinct things (see Križ 2015): An answer to (18b) increases the likelihood of a certain answer to (18a) without technically entailing it. Since even the proposition that all the doors were open is not technically strongly relevant to (18a), a theory of non-maximality based on a non-probabilistic notion of relevance has to assume that the value of $\nu_C(I)$ is not the actual QUD, but another question that is a good proxy for the actual QUD and makes at least some propositions in $[\phi]^{\nu_C, w}$ strongly relevant.

- (18) In the BANK ROBBERY scenario:
- a. **Actual QUD:** ‘Were you able to reach the safe?’
- b. **Issue parameter:** ‘Was there a plurality x of doors such that x provides a path to the safe and all doors in x were open?’

Given this setup, the crucial property of contexts like HOMETOWN or DRYING WALLS is the following: The salient issue $\nu_C(I)$ is such that there are many less fine-grained issues that are roughly as ‘informative’ for the actual QUD as $\nu_C(I)$

itself. The notion of a less fine-grained issue or **subquestion** is defined in (19). Intuitively, subquestions of an issue Q are obtained by unifying some cells in Q .

- (19) An issue Q' is a **subquestion** of Q iff every partition class of Q is a subset of a partition class of Q' .

For instance, if the actual QUD in the DRYING WALLS scenario is as in (20a), a plausible default value of the issue parameter is (20b), which is a good proxy for this QUD regardless of where one takes the cutoff point for ‘good ventilation’ to be. But, since closing one or two of the 50 windows will affect ventilation only slightly, the less fine-grained issue in (21) is an almost equally good proxy for the actual QUD.

- (20) a. **Actual QUD:** ‘Is the room well ventilated?’

b. **Issue parameter:** $Q_{\geq 49} =$

w_{50}, w_{49}	w_{48}, w_{47}	w_{46}, w_{45}	w_{44}, w_{43}	w_{42}, \dots
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(21) **Alternative issue 1:** $Q_{\geq 47} =$

$w_{50}, w_{49}, w_{48}, w_{47}$	$w_{46}, w_{45}, w_{44}, w_{43}$	w_{42}, \dots
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The issues $Q_{\geq 49}$ and $Q_{\geq 47}$ in (20b) and (21) both permit some degree of non-maximality, such that a single closed window does not affect the acceptability of *The windows are open*. But $Q_{\geq 49}$ and $Q_{\geq 47}$ disagree on whether two or three closed windows make a difference. The idea then is that if a sentence like *The windows are open* is pragmatically true under some, but not all of the subquestions under consideration (e.g., under $Q_{\geq 47}$, but not $Q_{\geq 49}$), it will have borderline status.

How does this relate to the Sorites paradox? Let \sim_C be the relation that holds between a question Q and a subquestion Q' of Q if Q' is an almost equally good proxy for the real QUD in C as Q . Put differently, an answer to Q' is ‘almost as useful’ as an answer to Q to someone trying to answer the real QUD in C . Then the basic idea will be that vagueness arises in a context C if there is a sequence Q_1, \dots, Q_n , where $Q_1 = v_C(I)$ and $Q_1 \sim_C Q_2, Q_2 \sim_C Q_3$ etc. up to Q_n , but $Q_1 \not\sim_C Q_n$. To illustrate, given the issues $Q_{\geq 49}$ and $Q_{\geq 47}$ from (20b) and (21) above and the even less fine-grained issue $Q_{\geq 43}$ in (22), one could have $Q_{\geq 49} \sim_C Q_{\geq 47}, Q_{\geq 47} \sim_C Q_{\geq 43}$ and $Q_{\geq 49} \not\sim_C Q_{\geq 43}$.

I will take this failure of transitivity to be at the heart of the Sorites paradox: The relation $Q_{\geq 49} \sim_C Q_{\geq 47}$ lets us infer the acceptability of *The windows are open* in w_{48} from its acceptability in w_{49} . Several further steps from acceptability in w_{i+1} to acceptability in w_i can be licensed in a similar manner by other subquestions standing in the \sim_C relation. But since $Q_{\geq 49} \not\sim_C Q_{\geq 43}$ holds, the relation \sim_C will not take us directly from acceptability in w_{49} to, say, acceptability in w_{43} .

(22) **Alternative issue 2:** $Q_{\geq 43} =$

$w_{50}, w_{49}, w_{48}, w_{47}, w_{46}, w_{45}, w_{44}, w_{43}$	w_{42}, \dots
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Since this generalization relates vagueness to the presence of multiple subquestions, it will also account for the lack of vagueness in the BANK ROBBERY scenario: In this scenario, $v_C(I)$ has just two partition classes and therefore cannot have any non-trivial subquestions. In contrast, Burnett’s (2017) examples make it easy to accommodate issues with many partition classes. In sum, the idea is to reduce vagueness in non-maximal predication to structural properties of $v_C(I)$ and \sim_C .

In this paper, my goal is not to formalize the relation \sim_C , but merely to show how non-maximality can be integrated into a general theory of vagueness that relies on non-transitive similarity relations between the values of a contextual parameter. I will first show how this works for *tall* and then return to non-maximality in Section 6.

5 Super-/subvaluationist strict/tolerant semantics

The approach to vagueness I will adopt, due to Cobreros et al. (2012a), is a version of the **tolerant/strict framework** (Cobreros et al. 2012b). In this theory, a sentence ϕ has **strict** and **tolerant truth conditions**, determined by two distinct interpretation functions that diverge when vague predicates are involved. A sentence has borderline status if it is tolerantly, but not strictly true; the Sorites paradox is accounted for by introducing a non-classical inference relation that permits us to shift from a strict interpretation of the premise to a tolerant interpretation of the conclusion. My implementation of this framework will differ from Cobreros et al. 2012a because 1) I view the Sorites argument as a context-relative inference pattern rather than a logically valid one, and 2) for compatibility with the issue-based literature, the account has to be translated into a possible-worlds framework.

5.1 Tolerance relations

We start by formalizing the idea that a context may leave the value of a contextual parameter underspecified. Let \mathcal{C} be the set of those contextual parameters that are associated with scales. Then for any parameter $d \in \mathcal{C}$, a context C provides a partial ordering $\leq_{C,d}$ among its possible values. \mathcal{C} will include a threshold parameter d_P for any vague lexical predicate P (e.g. *tall*), but also includes the issue parameter I , since issues can be partially ordered by the subquestion relation.

The basic idea will be that for any parameter d in \mathcal{C} , a context C provides a default value $v_C(d)$ as before—but in addition, it provides a set of alternative values that count as “close enough” to $v_C(d)$ in the ordering $\leq_{C,d}$. To formalize this notion of being “close enough”, we introduce the notion of a **tolerance relation**:

- (23) A **tolerance relation** on a partial ordering \leq is a relation \sim such that 1) \sim is reflexive and 2) \sim satisfies the following **convexity** condition: If $x \leq y \leq z$

and $x \sim z$, then $x \sim y$ and $y \sim z$.⁷

- (24) For any parameter $d \in \mathcal{C}$, a context C provides a partial ordering $\leq_{C,d}$ on the set of possible values of d , as well as a tolerance relation $\sim_{C,d}$ on $\leq_{C,d}$.

Intuitively, the tolerance relation $\sim_{C,d}$ for a parameter d holds of two potential parameter values x, y if the difference between x and y can be neglected for the purposes of C . For instance, the judgment that if someone of height d counts as *tall*, so does someone of height $d - 1\text{cm}$, can be modeled by the following context $C_{1\text{cm}}$:

- (25) $v_{C_{1\text{cm}}}(d_{\text{tall}}) = 180\text{cm}; d' \sim_{C_{1\text{cm}}, d_{\text{tall}}} d''$ iff $|d' - d''| \leq 1\text{cm}$

Given a context C with a family \sim_C of tolerance relations, there are many different ways of selecting a set of thresholds for the different scalar parameters, or equivalently a valuation function, within the bounds set by these relations:

- (26) Let C be a context. A valuation function v is **compatible** with C iff for all parameters $d \in \mathcal{C}$, $v_C(d) \sim_{C,d} v(d)$.

For instance, consider the context $C_{1\text{cm}}$ defined in (25). For any valuation function v compatible with $C_{1\text{cm}}$, $v(d_{\text{tall}})$ is in the interval $[179\text{cm}, 181\text{cm}]$. The basic idea will then be that, if some, but not all of the valuation functions v compatible with $C_{1\text{cm}}$ are such that Peter's height exceeds $v(d_{\text{tall}})$, then the sentence *Peter is tall* will have borderline status in $C_{1\text{cm}}$. The next step is to formally implement this idea.

5.2 Strict and tolerant truth and falsity conditions

I follow Cobreros et al. (2012a) in taking vagueness to be a post-compositional phenomenon (*contra* Cobreros et al. 2012b). Semantic composition proceeds on the basis of a valuation function, yielding clear-cut truth conditions. In the absence of plurals, the pragmatic truth conditions coincide with the semantic ones:

- (27) $\llbracket \textit{Peter is tall} \rrbracket_{\mathbf{p}}^{v,w} = \llbracket \textit{Peter is tall} \rrbracket^{v,w} = 1$ iff $\text{HEIGHT}_w(\mathbf{Peter}) \geq v(d_{\text{tall}})$

The strict and tolerant truth conditions in a context C are obtained by universal and existential quantification, respectively, over the valuation functions compatible with C . (For completeness, I define strict and tolerant falsity conditions in a symmetric way.) So if ϕ is true under some, but not all of these valuation functions, it is tolerantly, but not strictly true. (29) illustrates this for the context $C_{1\text{cm}}$ from (25).

⁷ Cobreros et al. (2012b,a) additionally require tolerance relations to be symmetric. Here I follow Burnett's (2017) extension of their framework in permitting non-symmetric relations (this will become relevant in the plural domain). The idea that tolerance relations must satisfy a convexity condition relative to a degree scale is taken from Burnett 2017, although in her system the structure of degree scales is derived from the strict/tolerant system rather than taken as primitive.

- (28) a. $\llbracket \phi \rrbracket_{\mathbf{t}}^{C,w} = 1$ (i.e. ϕ is **tolerantly true** wrt. C, w) iff $\llbracket \phi \rrbracket_{\mathbf{p}}^{v,w} = 1$ for some valuation function v compatible with C .
- b. $\llbracket \phi \rrbracket_{\mathbf{t}}^{C,w} = 0$ (i.e. ϕ is **tolerantly false** wrt. C, w) iff $\llbracket \phi \rrbracket_{\mathbf{p}}^{v,w} = 0$ for some valuation function v compatible with C .
- c. $\llbracket \phi \rrbracket_{\mathbf{s}}^{C,w} = 1$ (i.e. ϕ is **strictly true** wrt. C, w) iff $\llbracket \phi \rrbracket_{\mathbf{p}}^{v,w} = 1$ for every valuation function v compatible with C .
- d. $\llbracket \phi \rrbracket_{\mathbf{s}}^{C,w} = 0$ (i.e. ϕ is **strictly false** wrt. C, w) iff $\llbracket \phi \rrbracket_{\mathbf{p}}^{v,w} = 0$ for every valuation function v compatible with C .
- (29) a. $\llbracket \textit{Peter is tall} \rrbracket_{\mathbf{t}}^{C_{1\text{cm}},w} = 1$ iff $\text{HEIGHT}_w(\mathbf{Peter}) \geq 179\text{cm}$
- b. $\llbracket \textit{Peter is tall} \rrbracket_{\mathbf{s}}^{C_{1\text{cm}},w} = 1$ iff $\text{HEIGHT}_w(\mathbf{Peter}) \geq 181\text{cm}$

5.3 Accounting for the paradox

Cobrerros et al. (2012a) make two assumptions about the pragmatics of the strict/tolerant distinction. First, for the purposes of truth-value judgments, a sentence that is tolerantly, but not strictly true has borderline status. The second assumption concerns reasoning: For an inference to be judged acceptable, it is sufficient if strict truth of the premises guarantees tolerant truth of the conclusion. This assumption is motivated by the conditional inferences in a Sorites argument, such as (30). The inference relation \models_C underlying these judgments is formally defined in (31). Unlike the inference relation \models_{st} from Cobrerros et al. 2012a, this relation is relativized to a context C , because my goal is to model a notion of acceptable inference in a context, rather than logical entailment. This distinction becomes crucial in the plural domain, where the acceptability of the Sorites argument is context-dependent.

(30) *Peter is tall (for a German man) if his height is 181cm. \models Peter is tall (for a German man) if his height is 180cm.*

(31) Given a context C : $\phi \models_C \psi$ iff $\{w: \llbracket \phi \rrbracket_{\mathbf{s}}^{C,w} = 1\} \subseteq \{w: \llbracket \psi \rrbracket_{\mathbf{t}}^{C,w} = 1\}$.

How does the strict/tolerant framework account for inferences of the type (30)? Let us assume for simplicity that *if his height is ncm* is a strict conditional quantifying over all worlds in which Peter's height is ncm . Then the strict truth conditions of the premise of (30), given in (32a), say that no degree that is within one $\sim_{C,d_{\text{tall}}}$ -step of the default threshold $v_C(d_{\text{tall}})$ may exceed 181cm. Given a non-trivial choice of $\sim_{C,d_{\text{tall}}}$, the default threshold $v_C(d_{\text{tall}})$ will then be below 181cm. For instance, this condition is met in the context $C_{1\text{cm}}$ defined in (25), where the default threshold is 180cm. The tolerant truth conditions for the conclusion of (30), given in (32b), merely require there to be some degree within one $\sim_{C,d_{\text{tall}}}$ -step of $v_C(d_{\text{tall}})$ that does not exceed 180cm. A context such as $C_{1\text{cm}}$ from (25) guarantees this inference,

because its tolerance relation permits deviations of up to 1cm.⁸ This accounts for the acceptability of classically invalid inferences like (30) in some contexts.

- (32) a. $\llbracket \text{If Peter's height is 180cm, Peter is tall} \rrbracket_s^{C,w}$
 $= 1$ iff $\forall v$ [v is a valuation function compatible with C
 $\rightarrow \llbracket \text{If Peter's height is 180cm, Peter is tall} \rrbracket_p^{v,w} = 1$]
 $= 1$ iff $\forall d$ [$v_C(d_{tall}) \sim_{C,d_{tall}} d$
 $\rightarrow \forall w' [\text{HEIGHT}_{w'}(\mathbf{Peter}) \geq 180\text{cm} \rightarrow \text{HEIGHT}_{w'}(\mathbf{Peter}) \geq d]$]
 $= 1$ iff $\forall d$ [$v_C(d_{tall}) \sim_{C,d_{tall}} d \rightarrow d \leq 180\text{cm}$]
- b. $\llbracket \text{If Peter's height is 179cm, Peter is tall} \rrbracket_t^{C,w}$
 $= 1$ iff $\exists v$ [v is a valuation function compatible with C
 $\wedge \llbracket \text{If Peter's height is 179cm, Peter is tall} \rrbracket_p^{v,w} = 1$]
 $= 1$ iff $\exists d$ [$v_C(d_{tall}) \sim_{d_{tall}} d \wedge d \leq 179\text{cm}$]

The use of the inference relation \models_C provides a new perspective on the Sorites argument in (11) as a whole. Given the context $C_{1\text{cm}}$ from (25), the premise in (11a) is strictly true, while the conclusion in (11c) is strictly false, conforming to intuition. But unlike in the classical setting, the inference relation $\models_{C_{1\text{cm}}}$ also validates all the conditional inferences matching the schema (11b). Nonetheless, the implausible inference in (33) is not derived, because $\models_{C_{1\text{cm}}}$ is not transitive. This failure of transitivity accounts for the seemingly paradoxical judgment that each individual step of the argument is acceptable, while the argument as a whole is not.

- (33) *If Peter's height is 185 cm, Peter is tall (for a German man). \models_C If Peter's height is 175 cm, Peter is tall (for a German man).*

Note that given this analysis, the paradox arises only in contexts C where the tolerance relation $\sim_{C,tall}$ is not transitive. Thus, we can have e.g. $185\text{cm} \sim_{C,tall} 184\text{cm}$, $184\text{cm} \sim_{C,tall} 183\text{cm}$ etc. all the way down to 175cm , but $185\text{cm} \not\sim_{C,tall} 175\text{cm}$. We will now extend this account of the paradox to the plural domain.

6 Strict and tolerant truth conditions for the issue parameter

Recall that the goal is to account for the acceptability of the reasoning in (34) in the DRYING WALLS context, while blocking the analogous inference in the

⁸ There is a technical complication here: The inference relation in (31) is based on a notion of entailment as inclusion between sets of worlds, but the strict and tolerant truth conditions in (32) do not depend at all on the world parameter, so that the inference in (30) is predicted to hold (or to fail) vacuously. This can be fixed by letting \models_C quantify over valuation functions, leaving only the scalar orderings and tolerance relations fixed. However, quantifying over *all* valuation functions will not work since for plural sentences, we can always find a valuation function that forces the strict and tolerant truth conditions of the sentence to coincide regardless of \sim_C (see Section 6.4 for discussion). The task of resolving this tension is left to future work.

BANK ROBBERY scenario. Given the simplified semantics of the metalinguistic predicate *acceptable* from Section 4, the relevant conditional statements have the truth conditions schematized in (35) under a valuation function v .

- (34) a. *If all the windows are open, **The windows are open** is acceptable.*
 b. For any $n > 2$, the following inference holds:
*If n of the windows are open, **The windows are open** is acceptable. \models If $n - 1$ of the windows are open, **The windows are open** is acceptable.*
 c. \therefore *If 2 of the windows are open, **The windows are open** is acceptable.*
- (35) $\llbracket \text{If } n \text{ of the windows are open, **The windows are open** is acceptable} \rrbracket^{v,w}$
 $= 1$ iff $\forall w [\exists x [\mathbf{windows}_w(x) \wedge |x| = n \wedge \mathbf{open}_w(x)] \rightarrow$
 $\forall p [p \in \llbracket \text{The windows are open} \rrbracket^{v,w} \wedge p \text{ strongly relevant to } v(I) \rightarrow p(w) =$
 $1]]$

The acceptability of the inference schema in (34b) is a puzzle for the issue-based approach in its standard form, because for any given issue, the truth conditions in (35) are clear-cut. To illustrate, let $v_C(I)$ be the issue $w_{\geq 49}$ defined in (20b), which separates w_{50} and w_{49} from w_{48} (recall that w_n is a world with n of the 50 windows open). Then $\llbracket \text{The windows are open} \rrbracket_{\mathbf{p}}^{v_C, w} = 1$ if $w \in \{w_{50}, w_{49}\}$, but $\llbracket \text{The windows are open} \rrbracket_{\mathbf{p}}^{v_C, w_{48}} \neq 1$. So even though C is a non-maximal context, the inference schema (34b) is predicted to fail in this context for $n = 49$. This reasoning generalizes to any other choice of $v_C(I)$ that requires more than two open windows. The issue-based approach therefore fails to distinguish between vague non-maximal contexts like DRYING WALLS and non-vague ones like BANK ROBBERY.

6.1 Vague non-maximal contexts

The underlying problem is that while the issue-based approach provides a plausible account of the interpretation of plurals relative to a valuation function, we need a way of extending it to contexts compatible with multiple valuation functions. Strict/tolerant semantics provides a natural way of filling this gap.

Recall that we derived strict and tolerant truth conditions in a context C by quantifying over the valuation functions compatible with C . In the case of *tall*, these functions differ in the value of the threshold parameter d_{tall} . To extend the framework to non-maximality, we need to include the issue parameter I in the set \mathcal{C} of parameters subject to vagueness. The valuation functions compatible with a given context can then differ in the value they assign to I , in line with the intuition that vague non-maximal contexts involve ‘vague questions’. Given the framework of Section 5, a context must then provide a tolerance relation between issues.

How can we characterize this relation pre-theoretically? In Section 4, I informally suggested that there is a relation \sim_C such that $Q \sim_C Q'$ holds iff Q' is a subquestion

of Q such that an answer to Q' would be ‘almost as useful’ as Q itself to someone trying to answer the actual QUD. For instance, in the DRYING WALLS scenario, where the actual QUD is something like ‘How good is the ventilation in the room?’, this relation arguably holds between $Q_{\geq 49}$ in (20b) and $Q_{\geq 47}$ in (21). We now take this relation to be the tolerance relation $\sim_{C,I}$ for the issue parameter. The restriction to subquestions can be expressed as a general constraint on contexts as follows:

(36) If $Q \sim_{C,I} Q'$, then Q' is a subquestion of Q .

As already suggested in Section 4, we can then make sense of the plural Sorites paradox and its context-dependency by observing that $\sim_{C,I}$ may be transitive in some contexts, but does not have to be transitive.

6.2 Strict and tolerant truth in the plural case

Assuming that the pragmatic truth value $\llbracket \phi \rrbracket_{\mathbf{p}}^{v,w}$ of a plural sentence ϕ is as defined in (7), its strict and tolerant truth conditions are as follows.⁹

- (37) a. $\llbracket \phi \rrbracket_{\mathbf{t}}^{C,w} = 1$ iff for some valuation function v compatible with C , $\llbracket \phi \rrbracket_{\mathbf{p}}^{v,w} = 1 = 1$ iff for some Q such that $v_C(I) \sim_{C,I} Q$, every proposition $p \in \llbracket \phi \rrbracket^{v,w}$ that is strongly relevant to Q is such that $p(w) = 1$.
- b. $\llbracket \phi \rrbracket_{\mathbf{s}}^{C,w} = 1$ iff for every valuation function v compatible with C , $\llbracket \phi \rrbracket_{\mathbf{p}}^{v,w} = 1 = 1$ iff for every Q such that $v_C(I) \sim_{C,I} Q$, every proposition $p \in \llbracket \phi \rrbracket^{v,w}$ that is strongly relevant to Q is such that $p(w) = 1$.

The tolerant truth conditions in (37a) say that there is some Q standing in the $\sim_{C,I}$ -relation to $v_C(I)$ such that all the propositions from $\llbracket \phi \rrbracket^{v,w}$ that are strongly relevant to Q are true. Since any such Q is a subquestion of $v_C(I)$, the propositions that are strongly relevant to Q are a subset of those strongly relevant to $v_C(I)$. For instance, consider a context C_{WALLS} conforming to the DRYING WALLS scenario that has the following properties (see (20b), (21), (22) for the issues $Q_{\geq 49}$, $Q_{\geq 47}$ and $Q_{\geq 43}$):

(38) $v_{C_{\text{WALLS}}}(I) = Q_{\geq 49}$, $Q_{\geq 49} \sim_{C_{\text{WALLS}},I} Q_{\geq 47} \sim_{C_{\text{WALLS}},I} Q_{\geq 43}$, $Q_{\geq 49} \not\sim_{C_{\text{WALLS}},I} Q_{\geq 43}$

Since every cell of $Q_{\geq 47}$ is the union of two cells of $Q_{\geq 49}$, any proposition that is strongly relevant to $Q_{\geq 47}$ is also strongly relevant to $Q_{\geq 49}$. Hence, the tolerant truth conditions of a sentence ϕ in C_{WALLS} can never be stronger than its truth conditions under the valuation function $v_{C_{\text{WALLS}}}$, where only the issue $Q_{\geq 49}$ is considered. But crucially, they may be weaker. The proposition $p_{49} = [\lambda w'. \text{at least } 49$

⁹ In the text, I make the simplifying assumption that the only contextual parameter relevant for this sentence is Q . This is arguably not really the case since the predicate *open* may itself be imprecise.

windows_w are open in w'] is strongly relevant to $Q_{\geq 49}$, but not to $Q_{\geq 47}$. However, $p_{47} = [\lambda w'. \text{at least 47 of the 50 windows}_w \text{ are open in } w']$ is strongly relevant to both issues. This gives us the following truth conditions:

- (39) $\llbracket \textit{The windows are open} \rrbracket_{\mathbf{p}}^{v_C, w} = 1$ iff at least 49 of the windows are open
 (40) For a valuation function v' with $v'(I) = Q_{\geq 47}$: $\llbracket \textit{The windows are open} \rrbracket_{\mathbf{p}}^{v', w} = 1$ iff at least 47 of the windows are open

The tolerant truth conditions in (37a) are met if $\llbracket \textit{The windows are open} \rrbracket_{\mathbf{p}}^{v, w} = 1$ for at least one valuation function v compatible with C_{WALLS} . Since v' in (40) is compatible with C_{WALLS} , this means that if 48 windows are open, *The windows are open* will be tolerantly true in C_{WALLS} , even though it is not true under $Q_{\geq 49}$. So the tolerant truth conditions of a sentence ϕ in a vague non-maximal context C are weaker than its truth conditions under the default valuation function. Specifically, this is the case if C provides a subquestion of the issue $v_C(I)$ that is ‘almost as useful’ for answering the overall QUD as $v_C(I)$ itself and makes fewer propositions in $\llbracket \phi \rrbracket^{v_C, w}$ relevant.

How does the presence of such subquestions affect the strict truth conditions? As it turns out, not at all. The condition in (37b) requires us to compute the pragmatic truth conditions under all the subquestions standing in the $\sim_{C, I}$ -relation to the default issue $v_C(I)$ and conjoin them. For instance, in the context C_{WALLS} described in (38), we would have to conjoin at least the pragmatic truth conditions ϕ has under the issue $v_{C_{\text{WALLS}}}(I) = Q_{\geq 49}$ and those it has under $Q_{\geq 47}$. As we have seen, a subquestion of $v_C(I)$ can never give rise to stronger truth conditions than $v_C(I)$ itself. So the strict truth conditions of ϕ in C cannot be stronger than its truth conditions under the valuation function v_C . Neither can they be weaker: Since $\sim_{C, I}$ must be reflexive and therefore relates $v_C(I)$ to itself, the strict truth conditions of ϕ in C must entail its truth conditions under v_C . The strict interpretation of a plural sentence in C is therefore identical to its interpretation under the default valuation function v_C .¹⁰

Note that the default issue $v_C(I)$ may already license non-maximality, in which case ϕ is strictly true in some non-maximal scenarios. So, while there is an analogy between the notion of tolerant truth and the notion of pragmatic truth given an issue, the present system (unlike Burnett 2017) builds a strict/tolerant truth definition on top of the issue-based pragmatic one, thus providing two ‘routes’ to non-maximality.

6.3 Deriving the paradox in the plural case

We now have all the necessary ingredients to account for the availability of the paradoxical inference pattern in (41) in the DRYING WALLS scenario. The strict and tolerant truth conditions of the crucial conditional statements are given in (42):

¹⁰ See Burnett 2017: ch. 7 for a derivation of the same result in a different framework.

- (41) a. *If all 50 of the windows are open, **The windows are open** is acceptable.*
 b. For any $n > 2$, the following inference holds:
*If n of the windows are open, **The windows are open** is acceptable. \models If $n - 1$ of the windows are open, **The windows are open** is acceptable.*
 c. \therefore *If 2 of the windows are open, **The windows are open** is acceptable.*
- (42) a. $\llbracket \text{If } n \text{ of the windows are open, **The windows are open** is acceptable} \rrbracket_s^{C,w}$
 $= 1$ iff $\forall Q[v_C(I) \sim_{C,I} Q \rightarrow \forall w[\exists x[\mathbf{windows}_w(x) \wedge |x| = n \wedge \mathbf{open}_w(x)]$
 $\rightarrow \forall p[p \in \llbracket \text{The windows are open} \rrbracket^{v_C,w} \wedge p \text{ strongly relevant to } Q \rightarrow p(w) =$
 $1]]]$
 $= 1$ iff $\forall w[\exists x[\mathbf{windows}_w(x) \wedge |x| = n \wedge \mathbf{open}_w(x)]$
 $\rightarrow \forall p[p \in \llbracket \text{The windows are open} \rrbracket^{v_C,w} \wedge p \text{ strongly relevant to } v_C(I) \rightarrow p(w) =$
 $1]]]$
- b. $\llbracket \text{If } n - 1 \text{ of the windows are open, **The windows are open** is acceptable} \rrbracket_t^{C,w}$
 $= 1$ iff $\exists Q[v_C(I) \sim_{C,I} Q \wedge \forall w[\exists x[\mathbf{windows}_w(x) \wedge |x| = n - 1 \wedge \mathbf{open}_w(x)]$
 $\rightarrow \forall p[p \in \llbracket \text{The windows are open} \rrbracket^{v_C,w} \wedge p \text{ strongly relevant to } Q \rightarrow p(w) =$
 $1]]]$

Since the strict truth conditions do not depend on $\sim_{C,I}$, (42a) simply requires that in any world with n open windows, all the propositions in $\llbracket \text{The windows are open} \rrbracket^{v_C,w}$ that are strongly relevant to $v_C(I)$ are true. In the context C_{WALLS} defined in (38), for any n with $1 \leq n \leq 49$, the proposition p_n (that n windows are open) is strongly relevant. The strict truth conditions in C require all these propositions to be true, which means at least 49 windows must be open.

But crucially, the tolerant truth conditions in (42b) involve existential quantification over a set of subquestions of $v_C(I)$ determined by the $\sim_{C,I}$ -relation. Given the context C_{WALLS} in (38), we have $v_C(I) \sim_{C,I} Q_{\geq 47}$ for the subquestion $Q_{\geq 47}$ in (21). Then the condition in (42b) is met for $n = 49$ if in any world with 48 open windows, all propositions in $\llbracket \text{The windows are open} \rrbracket^{v,w}$ that are strongly relevant to $Q_{\geq 47}$ are true. Since $Q_{\geq 47}$ puts w_{48} and w_{49} in the same partition class, the proposition p_{49} is not strongly relevant to $Q_{\geq 47}$. So in this context, the strict truth conditions in (42a) guarantee the tolerant truth conditions in (42b) for $n = 49$.

In sum, the tolerant truth conditions of a plural sentence may involve a higher degree of non-maximality than the strict ones. This is why in some contexts, strict truth of a conditional of the type (41b) for some n may guarantee its tolerant truth for $n - 1$, which accounts for the acceptability of the Sorites argument.

6.4 Non-vague non-maximal contexts

Let us now return to the BANK ROBBERY scenario, in which the paradox does not arise. The crucial property of this context is that the most salient issue is the yes/no

question $Q_{y/n} = \text{'Were all the doors on some path to the safe open?'}$

To see why this blocks vagueness, consider a context C such that $v_C(I) = Q_{y/n}$. Since the issue $Q_{y/n}$ has only two cells, it cannot have any proper subquestions. Given the constraint in (36), the only issue Q such that $Q_{y/n} \sim_{C,I} Q$ must then be $Q_{y/n}$ itself. But if $\sim_{C,I}$ does not relate $v_C(I)$ to any non-trivial subquestions, both the strict and the tolerant truth conditions of any sentence ϕ in C reduce to the condition that ϕ is true under v_C . So given an issue with just two partition classes, the strict and the tolerant truth conditions coincide and \models_C reduces to classical entailment.

Regardless of the precise choice of the issue $v_C(I)$, there are then two options: Either the conditional premise of the Sorites argument fails for some n , or $\llbracket \textit{The doors are open} \rrbracket_{\mathbf{p}}^{v_C, w} = 1$ even if just one or two doors are open in w , in which case the conclusion of the Sorites argument is unobjectionable in C . Either way, a context C with a binary issue will not give rise to the paradox, intuitively because the relevant part of $\sim_{C,I}$ is trivially transitive.

7 Conclusion and outlook

The present view of the relation between vagueness and non-maximality occupies a middle ground between the two existing approaches it is based on: Unlike most implementations of the issue-based approach, it attributes vagueness to plural sentences in some non-maximal contexts. But vagueness is taken to be less pervasive than in the earlier strict/tolerant analysis of non-maximality due to Burnett (2017). For Burnett, the strict truth conditions of plural sentences are always maximal, so that non-maximality always indicates a mismatch between the strict and tolerant truth conditions. Non-maximal plural predication should then be pragmatically on a par with borderline cases of vague predicates, which makes the problematic prediction that plural sentences are not fully acceptable in non-maximal scenarios.

The present framework avoids this problem by providing two different ways of deriving non-maximal truth conditions in a context C . First, non-maximality may come about if the strict and tolerant truth conditions diverge due to the structure of the tolerance relation $\sim_{C,I}$. This is the case in vague non-maximal contexts such as DRYING WALLS. But non-maximality can also arise due to the structure of the default issue $v_C(I)$, as in the BANK ROBBERY scenario. This second route to non-maximality does not require a mismatch between the strict and the tolerant truth conditions; as a consequence, it does not give rise to vagueness. In sum, the present framework introduces a distinction between two subtypes of non-maximality that may differ in their pragmatic impact. Whether there are any grammatical phenomena sensitive to this distinction, and how it relates to other proposals that assume two distinct sources of non-maximality for reasons independent of vagueness (e.g. Bar-Lev 2021) is an open question that I must leave to future research.

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